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Numerical Solution of Surface Waveguide Modes Using Transverse Field Components

C. G. WILLIAMS AND G. K. CAMBRELL

Abstract—**The computation of surface waveguide modes is facilitated by reducing the surface waveguide field problem to a conventional eigenvalue problem that has no spurious solutions. This is achieved by formulating the field problem in terms of transverse field components and by using impedance boundary conditions on an auxiliary boundary with a specified value of the exterior cutoff wavenumber.**

Introduction

In many field problems of practical interest, the region being considered is of infinite extent. A numerical method [1]–[3] which combines integral and differential equation approaches is found to be effective in increasing computational efficiency and accuracy. A further application of the method is described here, namely, the computation of surface waveguide modes. When formulated in terms of transverse field components, this is a two-dimensional exterior eigenvalue problem.

Selection of Field Components

A surface waveguide is essentially an inhomogeneous waveguide without a closed boundary. The wave equation describing the propagation in an inhomogeneous waveguide can be expressed in terms of two field components, which are usually taken to be the longitudinal components, E_z and H_z. (A field dependence of $\exp[j(\omega t - \beta z)]$ is assumed throughout.) However, as pointed out by Gelder [4], this choice leads to a generalized eigenvalue problem which, for a specified angular frequency ω, is nonlinear in the eigenvalue β^2. If the phase velocity ω/β is specified instead, a conventional problem with eigenvalue ω^2 is obtained, but the solutions include spurious nonsurface modes. This is because the exterior field of a surface mode decays exponentially corresponding to an imaginary exterior cutoff wavenumber k_A, that is, $k_A^2 = k_0^2 - \beta^2 = \omega^2\mu_0\epsilon_0 - \beta^2$ is negative for a surface mode, whereas the specification of ω/β is insufficient to determine k_A^2. On the other hand, for a specified value of k_A^2, use of the transverse components [4], E_x and E_y, or H_x and H_y, leads to a conventional eigenvalue problem with eigenvalue ω^2 which has no spurious solutions.

Problem Formulation

The cross section of a typical surface waveguide is shown in Fig. 1. The rectangular dielectric rod (permittivity ϵ) is enclosed within an auxiliary boundary C which divides all space into an interior region R

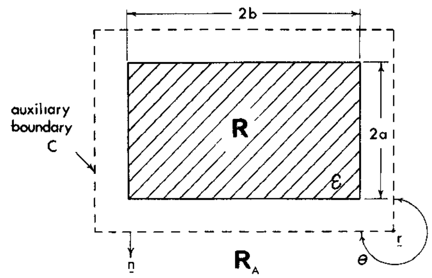

Fig. 1. Cross section of a rectangular dielectric rod surface waveguide.

and a homogeneous exterior region R_A. The transverse magnetic field satisfies the differential equation [5]

$$\nabla_t\left[\frac{1}{\mu}\nabla_t\cdot(\mu H_t)\right] - \epsilon\nabla_t \times \left[\frac{1}{\epsilon}(\nabla_t \times H_t)\right] = (\beta^2 - \omega^2\mu\epsilon)H_t. \quad (1)$$

Assuming uniform permeability μ_0, it is convenient to rearrange (1) into the following component form:

$$-(\nabla_t^2 + k_A^2)H_x - \frac{1}{\epsilon}\frac{\partial\epsilon}{\partial y}\left(\frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial x}\right) = \omega^2\mu_0(\epsilon - \epsilon_0)H_x \quad (2)$$

$$-(\nabla_t^2 + k_A^2)H_y = \frac{1}{\epsilon}\frac{\partial\epsilon}{\partial x}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = \omega^2\mu_0(\epsilon - \epsilon_0)H_y \quad (3)$$

which reduces to

$$-(\nabla_t^2 + k_A^2)H_t = 0 \quad (4)$$

in the homogeneous exterior region R_A. Although (1) is not self-adjoint, it can be solved in R by such conventional techniques as the method of moments [6]. For example, projecting both sides of (2) and (3) onto the space spanned by a set of testing functions $W_i(x,y)$ yields

$$\iint_R \left\{\frac{\partial W_i}{\partial x}\frac{\partial H_x}{\partial x} + \frac{\partial W_i}{\partial y}\frac{\partial H_x}{\partial y} - k_A^2 W_i H_x - \frac{1}{\epsilon}\frac{\partial\epsilon}{\partial y}W_i\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\right\} dA - \oint_C W_i \frac{\partial H_x}{\partial n}\, ds = \omega^2 \iint_R \mu_0(\epsilon - \epsilon_0)W_i H_x\, dA \quad (5)$$

$$\iint_R \left\{\frac{\partial W_i}{\partial x}\frac{\partial H_y}{\partial x} + \frac{\partial W_i}{\partial y}\frac{\partial H_y}{\partial y} - k_A^2 W_i H_y - \frac{1}{\epsilon}\frac{\partial\epsilon}{\partial x}W_i\left(\frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial x}\right)\right\} dA - \oint_C W_i \frac{\partial H_y}{\partial n}\, ds = \omega^2 \iint_R \mu_0(\epsilon - \epsilon_0)W_i H_y\, dA \quad (6)$$

where n is the outward normal. In addition, the transverse field components must also satisfy (4) in the homogeneous exterior region R_A. Hence the trial values of the transverse field H_{tC} and its outward derivative $\partial H_{tC}/\partial n$ on the auxiliary boundary cannot be independent. The compatibility condition which links them is found by applying Green's theorem to (4) to yield the integral equation

$$H_{tC}(r) = \frac{1}{\theta}\oint_C \left\{H_{tC}(r_0)\frac{\partial}{\partial n}K_0(k\,|\,r - r_0\,|) - K_0(k\,|\,r - r_0\,|)\frac{\partial H_{tC}}{\partial n}(r_0)\right\} ds_0 \quad (7)$$

where $k = (-k_A^2)^{1/2}$, $K_0(k\,|\,r - r_0\,|)$ is a modified Bessel function [Green's function for (4)], θ is the exterior angle in radians between the tangents on each side of the point r on C, and it is understood that

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The authors are with the Department of Electrical Engineering, Monash University, Clayton, Vic., Australia.

the principal value of the integral is calculated. The integral equation (7) is a boundary impedance constraint on the trial magnetic field in (5) and (6) which precisely represents the effect of the exterior free-space region R_A.

In order to proceed with the method of moments [(5) and (6)], it is necessary to express $\partial \boldsymbol{H}_{tC}/\partial n$ in (7) in terms of $\boldsymbol{H}_{tC}$. This can be achieved by a separate application of the method of moments to (7), which leads to the matrix relation

$$\psi_i = \sum_{j \text{ (over } C)} A_{ij}(k_A^2)\phi_j \tag{8}$$

where $\partial \boldsymbol{H}_{tC}/\partial n$ and $\boldsymbol{H}_{tC}$ are represented by the parameters ψ_i and ϕ_j, respectively. The boundary terms in (5) and (6) can therefore be approximated by the linear combinations

$$\oint_C W_i \frac{\partial H_x}{\partial n}\, ds = \sum_{j \text{ (over } C)} B_{ij}(k_A^2)\phi_j \tag{9}$$

$$\oint_C W_i \frac{\partial H_y}{\partial n}\, ds = \sum_{j \text{ (over } C)} C_{ij}(k_A^2)\phi_j. \tag{10}$$

Hence the application of the method of moments to (1) yields the matrix eigenvalue problem

$$\sum_{j \text{ (over } R \text{ and } C)} D_{ij}(k_A^2)\phi_j = \omega^2 \sum_{j \text{ (over } R \text{ and } C)} E_{ij}\phi_j \tag{11}$$

in which the value of k_A^2 is specified.

Example

The method was applied to the rectangular dielectric rod shown in Fig. 1. This simple example may be analyzed by placing the auxiliary boundary directly on the dielectric–air interface. The approximate field distributions and dispersion characteristics of the first few surface modes were obtained by solving (11) for a range of values of k_A^2. A coarse square mesh system on a 6 point $\times$ 5 point grid was used which resulted in matrices of order 60. Bilinear expansion functions and testing functions were used for (5) and (6) whereas point matching was used to reduce (7) to a matrix constraint. Fig. 2 shows the dispersion characteristic of the dominant ${}_0\mathrm{EH}_{11}$ surface mode for the case $b/a = 1.25$ and relative permittivity 2.5. Good agreement is obtained with the results of Schlosser and Unger [7].

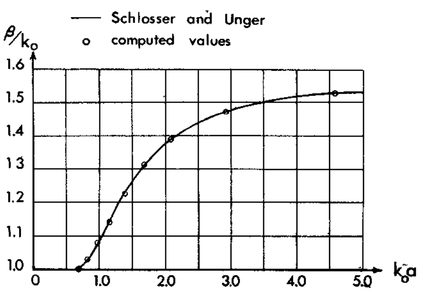

Fig. 2. Dispersion characteristic of the dominant ${}_0\mathrm{EH}_{11}$ surface mode of the dielectric rod ($b/a = 1.25$, $\epsilon/\epsilon_0 = 2.5$).

This method is currently being evaluated for obtaining dispersion characteristics of optical fibers and open-boundary structures which can support a quasi-TEM mode.

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Correction for Adapters in Microwave Measurements

A. UHLIR, JR., FELLOW, IEEE

***Abstract*—A measurement on a standard termination provides sufficient information for making corrected measurements through an adapter, if the dissipative loss of the adapter can be neglected. This approximation often gives more consistent results than calibration techniques that require highly reflecting standards.**

Introduction

Since it is impractical to develop measurement equipment for each of the transmission-line and connector types in common use, measurements are often carried out through passive reciprocal adapters (also called "transitions" or "transducers").

Adapters are designed to have low loss and low reflection. However, it is often necessary to apply computed corrections to achieve the desired accuracy. This paper presents a simple method for determining the corrections, based on the assumption that the adapter has negligible dissipative loss. The "primary" connector on the measuring apparatus will usually permit repeatable low-loss connections. The "secondary" connector type on the device under test may or may not permit consistent low-loss connections. If not, the simple method will give more valid results than the technique now commonly used with computer-controlled network analyzers.

Two examples will illustrate the intended applications of the method. In each case, suppose that a computer-controlled network analyzer system is available with 7-mm precision connectors: the "primary" connector system.

The first example is the measurement of devices with SMA connectors. Adapters from 7 mm to the "secondary" SMA connector, constructed with reasonable care, will have good conducting surfaces and negligible dielectric losses. It is probably more accurate to consider such an adapter to be dissipationless than it is to assume that low-loss connection of reference standards can be achieved consistently and repeatedly without excessive stress on the SMA connector.

The second example is the measurement of waveguide components with the 7-mm connector as a "primary" connector. This practice is simply an expedient to avoid setting up measurement apparatus in each of the numerous waveguide bands. The proposed method requires only a precision or sliding load as a reference standard in each waveguide type.

The discussion will consider a single frequency. It will be assumed that the network analyzer is linear or that stored calibrations to correct for nonlinearities have already been applied.

Present Calibration Technique

The present method is based upon carrying out, at the secondary connector, the same sort of calibration procedure as ordinarily used at the primary connector to determine corrections for residuals in the measurement system. For this purpose, reflection standards are required in the secondary connector system. Ordinarily, the standards used are a short circuit, a matched termination (which may be a sliding load), and an open circuit or an offset short circuit. A set of such standards is required for each connector type for which measurements are required.

The reflection coefficient is a bilinear function of the network analyzer output, so the computer program is the same for making corrected measurements at the secondary connector as at the primary connector.

When most of the measurements are to be made on highly reflecting devices, it is preferable to use three highly reflecting standards [1]. The calibration program must be modified in this case.

Simplified Method

In the simplified method, the network analyzer is calibrated for reflection measurements at its primary connector, by reference to reflection standards for the primary connector. Then the adapter is

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The author is with the Department of Electrical Engineering, Tufts University, Medford, Mass. 02155.